**FMN – IMA**
**Mathematics at Work**
**competition winner 2008**

# Jet Engines with Pythagoras

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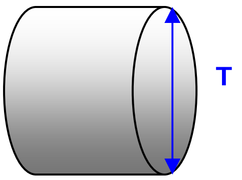
In our factory we make blisks (bladed discs) for the new Joint Strike Fighter and Eurofighter engines. These parts are required to spin at huge speeds and withstand considerable impact forces so clearly we have to know the parts are produced accurate to the design specifications. Due to the complex nature of the component some features are either impractical or impossible to measure. With an understanding of Pythagoras' Theorem and coordinate geometry, accessible points can be used to infer results for our 'illusive' features.

These techniques help us to check (amongst other things) thickness and blade movement.

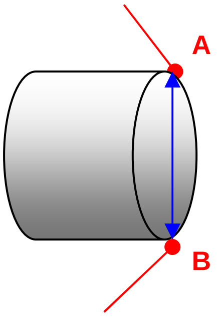
**Thickness:**

It is important to measure the thickness of the part in many locations to check that we are not cutting off metal that should be there, or leaving any on that doesn't need to be there. However, thickness sometimes needs to be known in locations that are not readily accessible for a traditional measurement instrument. To get round this we use some simple maths and a Coordinate Measuring Machine. We take 3D coordinate measurements on opposite sides of the section whose thickness we wish to calculate and use the Pythagoras theorem.

**For example**




We need to measure Thickness, T, but sometimes there are other bits of the part obstructing and preventing us from getting an accurate measurement.




We get round this by taking 3D coordinate measurements at the two opposite points (A & B) and work out the distance between these two points using Pythagoras' Theorem which gives us the thickness.

If Point A has co-ordinates $(x_1, y_1, z_1)$ and Point B has co-ordinates $(x_2, y_2, z_2)$ we can work out the distance between the points ($D_1$) using the following formula:

$$D_1 = \sqrt{(x_3^2 + y_3^2 + z_3^2)}$$

Where: $x_3 = (x_1 - x_2)$, $y_3 = (y_1 - y_2)$, $z_3 = (z_1 - z_2)$

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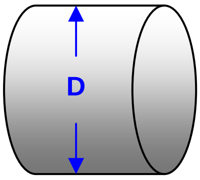
**Blade Movement:**

It is also important to measure how the part is 'moving' between manufacturing operations. For example if the blisk is being manually polished the blades should not move position at all, however if too much force is applied during the process it is possible that the blades could be shifted. Therefore we have to check.

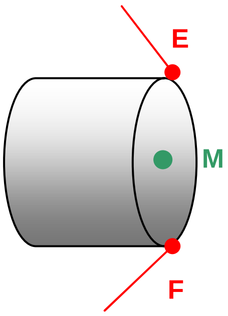
If you can imagine the movement we are looking at is so precise (in microns, which are one thousandth of a millimetre) that taking photos and visual inspection would not provide nearly enough detail for us to be sure that blades are not moving.

However, as with the thickness we can take 3D coordinates from points each side of the blade and by finding the midpoint and utilising Pythagoras' Theorem again, judge whether the blade is moving by determining if the midpoint has moved.

**For example**




If we wish to see how the blade has moved at section D between operations it is very hard to see from photos so instead we measure points on the section at opposite sides on the blade.




As with any two points we can work out the point exactly between them by using the midpoint formula.

If Point E has co-ordinates $(x_1, y_1, z_1)$and Point F has co-ordinates $(x_2, y_2, z_2)$ we can work out the coordinates of the midpoint (M) using the following formula:

$$M = (X_3, Y_3, Z_3)$$

Where: $X_3 = \dfrac{x_1 + x_2}{2}$, $Y_3 = \dfrac{y_1 + y_2}{2}$, $Z_3 = \dfrac{z_1 + z_2}{2}$

If we obtain the coordinates of M before and after each operation we can use Pythagoras' Theorem as we did for the thickness calculations to check the distance between the two points (M before and M after). The distance between those two points tells us how much M has moved and therefore how far section D has moved.