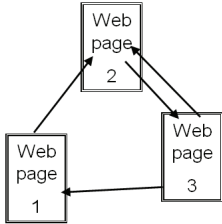


What do you study in Further Mathematics?

In Further Mathematics there are opportunities to study many different branches of mathematics; Pure, Decision, Statistics, Mechanics and Numerical mathematics. The internet search engine Google uses lots of ideas from several of these branches: Decision, Pure and Numerical Methods. You'll meet all the ideas on this sheet in these modules.

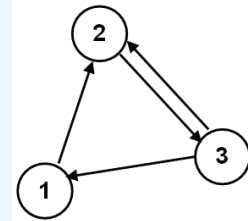


1. Let's take a simplified model of the internet, as in the diagram on the left; only three websites and five hyperlinks! The arrow from website 3 to website 1 means that there is a hyperlink from website 3 to website 1.

Webpage	Linked to from
1	3
2	1,3
3	2

2. Decision Maths 1 — Graphs

The internet can be represented as a diagram, called a graph, with arrows showing the hyperlinks between websites. The internet's graph will, of course, be absolutely enormous but there are ways of dealing with this, see below.



3. The earliest search engines simply counted the number of incoming links to measure a website's importance. Under this system, for the simplified model above, website 2 gets the top rank with websites 1 and 3 in joint second.

Webpage	Incoming Links
1	1
2	2
3	1

Google had the idea that when calculating the rank of a website, account should be taken of the ranking of the sites which link to it. If the 3 sites have rankings of x_1 , x_2 , and x_3 respectively, think of these as their voting power. Looking at the diagram, can you see how site 1 is receiving half of site 3's vote and how site 2 is receiving the other half of site 3's vote and all of site 1's vote?

The clever thing is that each sites' voting power is given by the votes that it receives, giving these equations.

$$\begin{aligned} x_1 &= 0.5x_3 \\ x_2 &= x_1 + 0.5x_3 \\ x_3 &= x_2 \end{aligned}$$

4. Further Pure Mathematics 1— Matrices

These equations can be written as a single matrix equation. One use of matrices is as a convenient way to solve simultaneous equations, particularly when there are many unknowns. The reason that our equations only have three unknowns is that we assumed a simplified internet model with only three websites.

$$\begin{pmatrix} 0 & 0 & 0.5 \\ 1 & 0 & 0.5 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

So the matrix for the whole internet is much bigger! However, it will have lots of zeros in it because most pairs of sites don't have links between them. This makes things easier to handle. Mathematicians research these so called 'sparse matrices'.

A solution of these equations is :

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ x_1 &= 0.2, x_2 = 0.4, x_3 = 0.4 \end{aligned}$$

5. Numerical Methods—Solutions of Equations

Search engines don't have much time to rank sites (think of how quickly your search is returned to you!). Iteration is used to approximate solutions quickly. On the right, T is the 3x3 matrix above.

x	Tx	T ² x	T ³ x	T ⁴ x	T ⁵ x	T ⁶ x	T ⁷ x	T ⁸ x	T ⁹ x	T ¹⁰ x
0.33	0.167	0.167	0.25	0.17	0.21	0.21	0.19	0.21	0.2	0.2
0.33	0.5	0.333	0.42	0.42	0.38	0.42	0.4	0.4	0.41	0.4
0.33	0.333	0.5	0.33	0.42	0.42	0.38	0.42	0.4	0.4	0.41

6. In each case below, try to calculate which of the sites below would receive the highest Google rank?

