OCR Further Pure 1

Complex Numbers

Section 3: Modulus and argument

Exercise

1. Find the modulus and argument of each of the following complex numbers:
   (i) $-2\sqrt{3}-2\text{i}$
   (ii) $1-3\text{i}$
   (iii) $-3+3\text{i}$

2. Write each complex number in the form $x + y\text{i}$.
   (i) $|z| = 3, \quad \arg z = \frac{\pi}{4}$
   (ii) $|z| = 6, \quad \arg z = \frac{2\pi}{3}$
   (iii) $|z| = 2, \quad \arg z = -\frac{\pi}{6}$

3. Given that $z = 1 + 2\text{i}$, find the modulus and argument of
   (i) z (ii) z^* (iii) $\frac{1}{z}$ (iv) $\frac{1}{z^*}$
   What do you notice?

4. Given that $w = 10\text{i}$ and $z = 1+\sqrt{3}\text{i}$, find the modulus and argument of each of w, z, wz, and $\frac{w}{z}$.
   What do you notice?

5. Draw an Argand diagram showing the set of points z for which the given condition is true.
   (i) $|z-1+\text{i}| = 1$
   (ii) $|z-2-3\text{i}| < 4$
   (iii) $1 < |z| < 2$
   (iv) $|z+1| = |z-3|$

6. Show that the equations $|z| = |z-2|$ and $|z-\text{i}| = |z-1|$ correspond to two straight lines in an Argand diagram. Find the value of z that satisfies both equations.

7. Represent the loci given by the equations $|z-3|=3$ and $|z|=|z-2|$ on the same Argand diagram and obtain the complex numbers corresponding to the points of intersection of these loci.

8. Draw an Argand diagram showing the set of points z for which the following conditions are true:

   (i) $\arg(z+2)=-\frac{2\pi}{3}$

   (ii) $\arg(z+2+\mathrm{i})=\frac{\pi}{2}$

   (iii) $\frac{\pi}{3}\le\arg(z-2)\le\pi$

9. Find a complex number z whose argument is $\frac{\pi}{4}$ and which satisfies the equation $|z+2+\mathrm{i}|=|z-4+\mathrm{i}|$.