OCR Further Pure 1

Complex Numbers

Section 1: Introduction to complex numbers

Exercise

1. Find the roots of the following equations:
   (i) $z^2 + 25 = 0$
   (ii) $4z^2 + 9 = 0$
   (iii) $z^2 - 2z + 2 = 0$
   (iv) $4z^2 + 4z + 5 = 0$

2. In each of the following cases find
   (a) $z_1 + z_2$ (b) $z_1 - z_2$ (c) $z_1 z_2$ (d) z_1^*
   (e) z_2^* (f) $z_1^* + z_2^*$ (g) $z_1^* - z_2^*$ (h) $z_1^* z_2^*$

   (i) $z_1 = 2 + 3i$; $z_2 = 1 - 2i$
   (ii) $z_1 = -2i$; $z_2 = 3 + i$
   What do you notice about the results?

3. Given that $z = (a + i)^4$ where a is real, find values for a such that
   (a) z is real,
   (b) z is wholly imaginary.

4. Simplify and write in the form $a + bi$
   (i) $\dfrac{1}{3+2i} + \dfrac{1}{3-2i}$
   (ii) $3 + i + \dfrac{4}{3-i}$
   (iii) $\dfrac{3}{1-i} - \dfrac{2i}{2+i}$

5. Find the values of p and q given that one root of the equation $z^2 + pz + q = 0$ is
   (i) $2 - i$
   (ii) $1 - 3i$
   (iii) $2i$
   (iv) $5 - 3i$

6. Given that $\dfrac{5}{a+bi} + \dfrac{2}{1+3i} = 1$, where a and b are real, find a and b.

7. Given that $a + bi$ is the conjugate of $(a + bi)^2$, find **all** possible pairs of values for a and b.

8. Find values for a and b given that:
   (i) $(a + b\mathrm{i})(2 + \mathrm{i}) \equiv a - 3\mathrm{i}$
   (ii) $(a + \mathrm{i})(4 - b\mathrm{i}) \equiv 3b + 2a\mathrm{i}$.

9. Find complex numbers z and w given that
   $(1 + \mathrm{i})\,z - \mathrm{i}w = \mathrm{i}z + (1 - \mathrm{i})w - 3\mathrm{i} = 6$