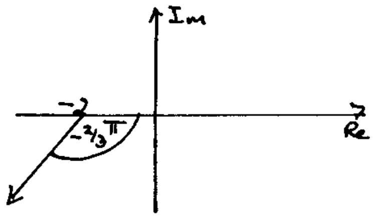
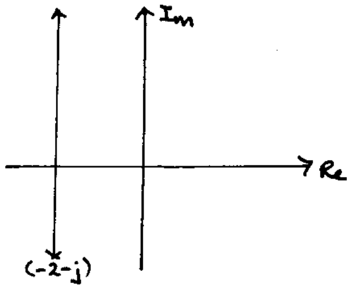
Further Pure 1

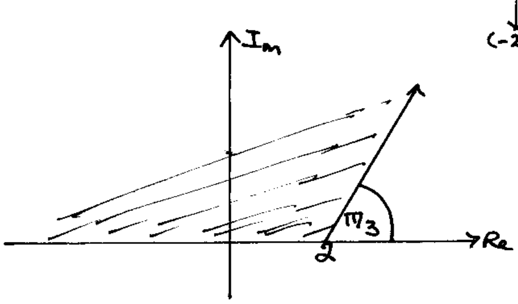
Complex Numbers
Exercise F

1 a)

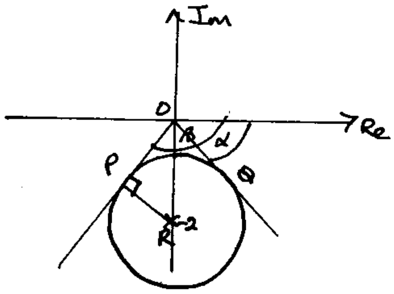


b)





2



$|z+2j| = 1$ rep circle centre $(0,-2)$ radius 1.

We will get the greatest and least values of arg z if we look at the tangents to the ⊙ from the origin i.e. OP and OQ

From △ OPR $\angle OPR = \pi/2$ (angle between tangent and radius)

$\cos \angle ORP = 1/2$

$\therefore \angle ORP = \pi/3$

$\angle POR = \pi/6$

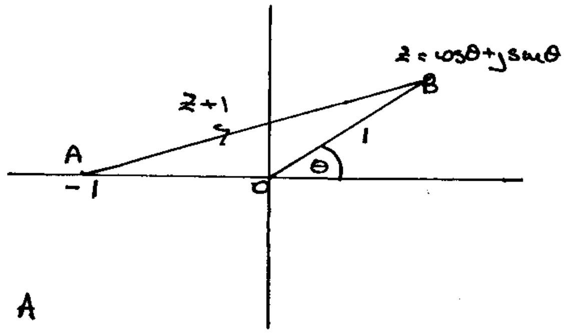
sim $\angle ROQ = \pi/6$

Least value of arg $z = \beta = -2\pi/3$

greatest value of arg $z = \alpha = -\pi/3$.

3 $z = \cos\theta + j\sin\theta$

$|z| = 1$



$z+1 = 1+\cos\theta + j\sin\theta$

$|z+1|$ rep. distance of z from A

From the diag

$|OB| = 1$ since this is $|z|$

$|OA| = 1$

$\triangle OAB$ is isosceles

$\angle A + \angle B = \theta$

i.e. $\angle A = \angle B = \theta/2$. $\therefore$ $\arg(z+1) = \theta/2$.

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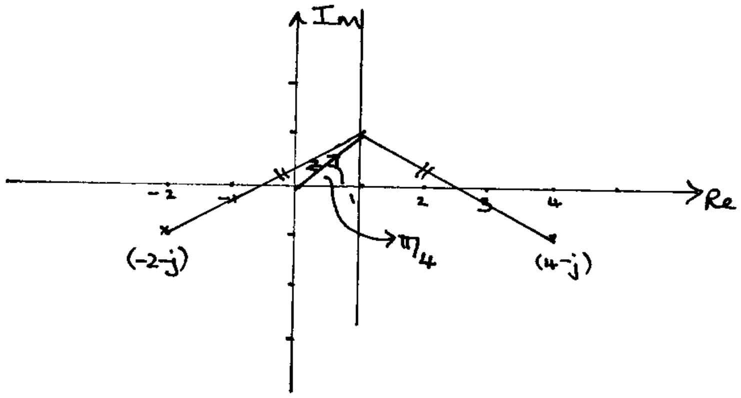
4 $|z+2+j| = |z-4+j|$ rep the $\perp^r$ bisector of the line joining $(-2-j)$ to $(4-j)$ $\therefore$ $Re(z) = 1$



For z to have an $\arg z = \pi/4$, $Im(z)$ must also be 1

$\therefore$ $z = 1+j$ satisfies the reqn^d conditions