Further Pure Mathematics 1

Complex Numbers

Glossary

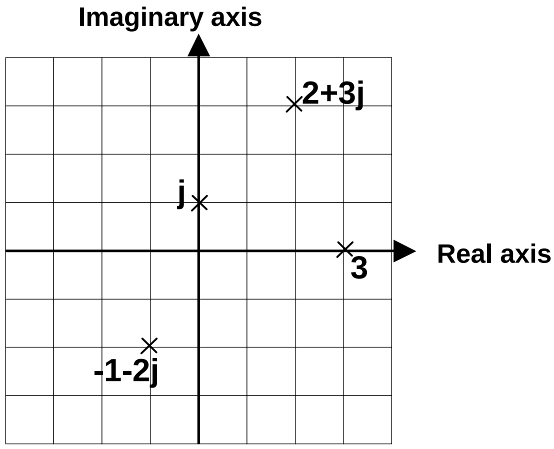
Argand diagram
The Argand diagram (or complex plane) is a geometrical representation of the complex numbers. The complex number $x + y\text{j}$ corresponds to the ordered pair of real numbers (x, y).

2 + 3j is represented by (2, 3)

-1 – 2j is represented by (-1, -2)

3 is represented by (3, 0)

j is represented by (0, 1)



Argument of a complex number
See the Principal argument of a complex number in this glossary.

Complex number
A complex number is a number of the form $x + y\text{j}$ where x and y are both real numbers. For example

$3 + 2\text{j}$

$-2 - \sqrt{2}\,\text{j}$

πj

are all complex numbers.

Complex plane
See Argand diagram in this glossary.

Conjugate (or complex conjugate)
The conjugate (sometimes called the complex conjugate) of a complex number $x + y\text{j}$ is the complex number $x - y\text{j}$. The complex conjugate of z is denoted by z^*.

Imaginary number
This is just another term for a complex number. Note the difference between this and a *purely* imaginary number (see elsewhere in this glossary).

Imaginary part of a complex number
For the complex number $z = x + y\text{j}$, y is called the imaginary part of the complex number. The imaginary part of a complex number z is denoted by Im(z).
For example: Im (5 + 4j) = 4
Im (-1) = 0
Im (-3j) = -3
Notice that the imaginary part of a complex number is itself a real number.

Integers
The set of integers consists of all the whole numbers (positive and negative), including zero.

j

j is a number whose defining property is that $\text{j}^2 = -1$. It is sometimes denoted by i.

Modulus-argument form of a complex number
For a complex number z with modulus r and principal argument θ, the modulus-argument (or polar) form of z is given by

$$z = r(\cos\theta + \text{j}\sin\theta)$$

Modulus of a complex number
The modulus of a complex number $z = x + y\text{j}$ is denoted by $|z|$ and defined to be $\sqrt{x^2 + y^2}$. Thus the modulus of a complex number is the distance from the point representing it to the origin in the Argand diagram.

Natural numbers
Also known as the counting numbers, the natural numbers are the numbers
1, 2, 3, 4…..

Polar form of a complex number
See the modulus-argument form of a complex number in this glossary.

Principal Argument of a complex number
The principal argument (or sometimes just argument) of a complex number z is denoted by $\arg(z)$. The argument of a non-zero complex number is defined as the angle it makes with the real axis measured anticlockwise from the real axis and chosen in such a way that $-\pi < \arg(z) \le \pi$. Positive real numbers

thus have argument 0 and negative real numbers have argument π. The argument of zero is undefined.

Pure imaginary number

A complex number is pure imaginary if its real part is zero. For example, 5j is pure imaginary, whereas 3 – 4j is not.

Real numbers

The set of all numbers whose imaginary part is zero. Real numbers include all rational and irrational numbers.

Real part of a complex number

For the complex number $z = x + y\text{j}$, x is called the real part of the complex number. The real part of a complex number is denoted by Re(z).

For example: Re (5 + 4j) = 5
Re (-1) = -1
Re (-3j) = 0

Rational numbers

A rational number is any number which can be expressed in the form $\frac{m}{n}$ where m and n are integers with $n \neq 0$.

For example: $\frac{27}{29}$ is rational

0.213 is rational because it can be expressed as $\frac{213}{1000}$

$\sqrt{2}$ is **not** rational

π is **not** rational.