Further Pure 1

Complex Numbers
Exercise G

8 (i) $z^3 + 6z^2 + 12z + 16 = 0$

Let $z = -4$

$(-4)^3 + 6(-4)^2 + 12 \times -4 + 16 = -64 + 96 - 48 + 16 = 0$

So $z = -4$ is a root. $\Rightarrow$ $z+4$ is a factor

$z^3 + 6z^2 + 12z + 16 = 0$

$(z+4)(z^2 + 2z + 4) = 0$

by inspection or polynomial division

$z^2 + 2z + 4 = 0 \Rightarrow z = \dfrac{-2 \pm \sqrt{(4 - 4\times 4)}}{2}$

$= \dfrac{-2 \pm \sqrt{-12}}{2}$

$= \dfrac{-2 \pm 2\sqrt{3}j}{2}$

$= -1 \pm \sqrt{3}j$

$\alpha = -4, \ \beta = -1 + \sqrt{3}j, \ \gamma = -1 - \sqrt{3}j.$

(ii) $\dfrac{1}{\beta} = \dfrac{1}{-1+\sqrt{3}j} = \dfrac{(-1-\sqrt{3}j)}{(-1+\sqrt{3}j)(-1-\sqrt{3}j)} = \dfrac{-1-\sqrt{3}j}{1+3} = -\frac{1}{4} - \frac{1}{4}\sqrt{3}j$

$\dfrac{1}{\gamma} = \dfrac{1}{-1-\sqrt{3}j} = \dfrac{(-1+\sqrt{3}j)}{(-1-\sqrt{3}j)(-1+\sqrt{3}j)} = \dfrac{-1+\sqrt{3}j}{1+3} = -\frac{1}{4} + \frac{1}{4}\sqrt{3}j$

(iii) $|\alpha| = |-4| = 4$

$\arg\alpha = \arg(-4) = \pi$

$|\beta| = |-1+\sqrt{3}j| = \sqrt{(1+3)} = 2$

$\arg\beta = \arg(-1+\sqrt{3}j) = \arctan\left(\frac{\sqrt{3}}{-1}\right) + \pi$

$= -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$

β is in the second quadrant

$|\gamma| = |-1-\sqrt{3}j| = \sqrt{(1+3)} = 2$

$\arg\gamma = \arg(-1-\sqrt{3}j) = \arctan\left(\frac{-\sqrt{3}}{-1}\right) - \pi$

$= \frac{\pi}{3} - \pi = -\frac{2\pi}{3}$

γ is in the third quadrant

(iv)