

EdExcel Further Pure 1

Complex Numbers

Section 3: Complex numbers and equations

Multiple Choice Test

- 1) The real numbers a and b that satisfy the equation

$$a + 3i - 2 + bi = b - i$$

are

- (a) $a = 2, b = -4$ (b) $a = -2, b = -4$
(c) $a = -2, b = -2$ (d) $a = 2, b = -2$
(e) I don't know

- 2) The real numbers a and b that satisfy the equation

$$a(2 - i) + b(1 + 4i) = 4 - 11i$$

are

- (a) $a = 2, b = -3$ (b) $a = 3, b = -2$
(c) $a = 2, b = -2$ (d) $a = 3, b = -3$
(e) I don't know

- 3) The square roots of the complex number $8 + 6i$ are

- (a) $1 - 3i$ and $-1 + 3i$ (b) $3 - i$ and $-3 + i$
(c) $1 + 3i$ and $-1 - 3i$ (d) $3 + i$ and $-3 - i$
(e) I don't know

- 4) The square roots of the complex number $5 - 12i$ are

- (a) $3 - 2i$ and $-3 + 2i$ (b) $3 + 2i$ and $-3 - 2i$
(c) $2 + 3i$ and $-2 - 3i$ (d) $2 - 3i$ and $-2 + 3i$
(e) I don't know

- 5) Which of the following groups of numbers could be the roots of a polynomial equation with real coefficients?

- (i) 3, 4, 5
(ii) $1 + i, 2, 4$
(iii) $1 - i, i, 1$
(iv) $2, 1 + i, 1 - i$

- (a) (i), (iii) and (iv) (b) (i) only
(c) (i) and (iv) (d) all of them
(e) I don't know

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6) $2 + i$ is a root of $z^3 - z^2 - 7z + 15 = 0$. The other roots are

- (a) $2 + i, 3$ (b) $2 - i, 3$
(c) $2 - i, -3$ (d) $2 + i, 2 - i$
(e) I don't know

7) The real root of $z^3 - 4z^2 + 14z - 20 = 0$ is 2. The other roots are

- (a) $2 + 3i, 2 - 3i$ (b) $-1 + 3i, -1 - 3i$
(c) $1 + 3i, 1 - 3i$ (d) $-2 + 3i, -2 - 3i$
(e) I don't know

8) $1 + 2i$ is a root of the cubic equation $z^3 + az^2 + bz + 5 = 0$.
The values of a and b are

- (a) $a = -1, b = 3$ (b) $a = 1, b = -1$
(c) $a = 1, b = 3$ (d) $a = -1, b = -1$
(e) I don't know

9) $-2 + i$ is a root of the equation $z^4 + 2z^3 - z^2 - 2z + 10 = 0$.
The other roots are

- (a) $2 - i, 1 + i, 1 - i$ (b) $-2 - i, 1 + i, 1 - i$
(c) $-2 - i, 2 - i, 2 + i$ (d) $-2 - i, 1 + 2i, 1 - 2i$
(e) I don't know

10) The equation $z^4 + z^3 + 2z^2 + 4z - 8 = 0$ has two real roots. The roots of the equation are

- (a) $-1, 2, 2i, -2i$ (b) $-1, 2, 1 + i, 1 - i$
(c) $1, -2, 1 + i, 1 - i$ (d) $1, -2, 2i, -2i$
(e) I don't know

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Solutions to Multiple Choice Test

1) The correct answer is (b)

$$a + 3i - 2 + bi = b - i$$

Equating imaginary parts: $3 + b = -1 \Rightarrow b = -4$

Equating real parts: $a - 2 = b \Rightarrow a - 2 = -4 \Rightarrow a = -2$

2) The correct answer is (b)

$$a(2 - i) + b(1 + 4i) = 4 - 11i$$

$$2a - ai + b + 4bi = 4 - 11i$$

Equating real parts: $2a + b = 4$ ①

Equating imaginary parts: $-a + 4b = -11$ ②

① + 2 × ② : $9b = -18$
 $b = -2, a = 3$

3) The correct answer is (d)

$$(a + bi)^2 = 8 + 6i$$

$$a^2 + 2abi - b^2 = 8 + 6i$$

Equating real parts: $a^2 - b^2 = 8$

Equating imaginary parts: $2ab = 6 \Rightarrow a = \frac{3}{b}$

Substituting: $\frac{9}{b^2} - b^2 = 8$

$$9 - b^4 = 8b^2$$

$$b^4 + 8b^2 - 9 = 0$$

$$(b^2 + 9)(b^2 - 1) = 0$$

$$b = \pm 1$$

When $b = 1, a = 3$

When $b = -1, a = -3$

The square roots of $5 - 12i$ are $3 + i$ and $-3 - i$.

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4) The correct answer is (a)

$$(a + bi)^2 = 5 - 12i$$

$$a^2 + 2abi - b^2 = 5 - 12i$$

$$\text{Equating real parts: } a^2 - b^2 = 5$$

$$\text{Equating imaginary parts: } 2ab = -12 \Rightarrow a = -\frac{6}{b}$$

$$\text{Substituting: } \frac{36}{b^2} - b^2 = 5$$

$$36 - b^4 = 5b^2$$

$$b^4 + 5b^2 - 36 = 0$$

$$(b^2 + 9)(b^2 - 4) = 0$$

$$b = \pm 2$$

$$\text{When } b = 2, a = -3$$

$$\text{When } b = -2, a = 3$$

The square roots of $5 - 12i$ are $3 - 2i$ and $-3 + 2i$.

5) The correct answer is (c)

Any complex roots of a polynomial equation with real coefficients occur in conjugate pairs. Therefore (ii) and (iii) cannot be the roots of a polynomial equation with real coefficients, since the complex roots do not occur in conjugate pairs. (iv) has two complex roots which are a conjugate pair, and (i) has all real roots, so both of these could be the roots of a polynomial equation with real coefficients.

6) The correct answer is (c)

Since $2 + i$ is a root, $2 - i$ is also a root.

So $(z - 2 - i)(z - 2 + i)$ is a factor of the equation.

$$(z - 2 - i)(z - 2 + i) = (z - 2)^2 + 1$$

$$= z^2 - 4z + 5$$

$$\text{So } z^3 - z^2 - 7z + 15 = (z^2 - 4z + 5)(z + 3)$$

So the third root is -3 .

The other two roots are $2 - i$ and -3 .

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7) The correct answer is (c)

2 is a root of the equation, so $z - 2$ is a factor.

$$z^3 - 4z^2 + 14z - 20 = 0$$

$$(z - 2)(z^2 - 2z + 10) = 0$$

The other two roots are the roots of the quadratic equation $z^2 - 2z + 10 = 0$

$$\begin{aligned} z &= \frac{2 \pm \sqrt{4 - 4 \times 1 \times 10}}{2} \\ &= \frac{2 \pm \sqrt{-36}}{2} \\ &= \frac{2 \pm 6i}{2} \\ &= 1 \pm 3i \end{aligned}$$

8) The correct answer is (a)

$$(1 + 2i)^2 = 1 + 4i - 4 = -3 + 4i$$

$$(1 + 2i)^3 = (-3 + 4i)(1 + 2i) = -3 - 2i - 8 = -11 - 2i$$

Substituting into $z^3 + az^2 + bz + 5 = 0$:

$$-11 - 2i + a(-3 + 4i) + b(1 + 2i) + 5 = 0$$

Equating real parts: $-11 - 3a + b + 5 = 0 \Rightarrow 3a - b = -6$

Equating imaginary parts: $-2 + 4a + 2b = 0 \Rightarrow 2a + b = 1$

Adding: $5a = -5 \Rightarrow a = -1, b = 3$

9) The correct answer is (b)

$-2 + i$ is a root, so $-2 - i$ is a root

so $(z + 2 - i)(z + 2 + i)$ is a factor.

$$(z + 2 - i)(z + 2 + i) = (z + 2)^2 + 1$$

$$= z^2 + 4z + 5$$

$$z^4 + 2z^3 - z^2 - 2z + 10 = (z^2 + 4z + 5)(z^2 - 2z + 2)$$

The other two roots are the roots of the quadratic equation $z^2 - 2z + 2 = 0$.

$$\begin{aligned} z &= \frac{2 \pm \sqrt{4 - 4 \times 1 \times 1}}{2} \\ &= \frac{2 \pm \sqrt{-4}}{2} \\ &= \frac{2 \pm 2i}{2} \\ &= 1 \pm i \end{aligned}$$

The other roots are $-2 - i, 1 + i$ and $1 - i$.

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10) The correct answer is (d)

$$f(z) = z^4 + z^3 + 2z^2 + 4z - 8$$

$$f(1) = 1 + 1 + 2 + 4 - 8 = 0$$

$$f(-2) = 16 - 8 + 8 - 8 - 8 = 0$$

so $(z - 1)$ and $(z + 2)$ are factors.

$$(z - 1)(z + 2) = z^2 + z - 2$$

$$z^4 + z^3 + 2z^2 + 4z - 8 = (z^2 + z - 2)(z^2 + 4)$$

The roots of $z^2 + 4 = 0$ are $2i$ and $-2i$.

So the roots of the equation are $1, -2, 2i$ and $-2i$.