

EdExcel Further Pure 1

Complex Numbers

Section 3: Complex numbers and equations

Exercise

1. Write in the form $f(z) = 0$, where $f(z)$ is a polynomial of degree 4 with real coefficients, the equation having $(3 + i)$ and $(1 + 3i)$ as two of its roots.
2. Find the real root of the equation $z^3 + z + 10 = 0$ given that one complex root is $1 - 2i$.
3. Given that $1 + i$ is a root of the equation $z^3 - 2z + k = 0$ find the value of k and the other two roots.
4. Show that $z = -1 + i$ is a root of the equation $z^4 - 2z^3 - z^2 + 2z + 10 = 0$ and find the other roots.
5. Given that one root of the equation $z^3 + az + b = 0$ is $p + qi$, where a and b are real and b is not zero, prove that
 - (i) $3p^2 - q^2 = -a$
 - (ii) $2p(p^2 + q^2) = b$
 - (iii) p is a root of the equation $8x^3 + 2ax - b = 0$
6. By writing $(a + bi)^2 = 3 - 4i$ find values for a and b and hence find the square roots of $3 - 4i$.
7. Given that $a + bi$ is the conjugate of $(a + bi)^2$, find **all** possible pairs of values for a and b .
8. Find values for a and b given that:
 - (i) $(a + bi)(2 + i) \equiv a - 3i$
 - (ii) $(a + i)(4 - bi) \equiv 3b + 2ai$.