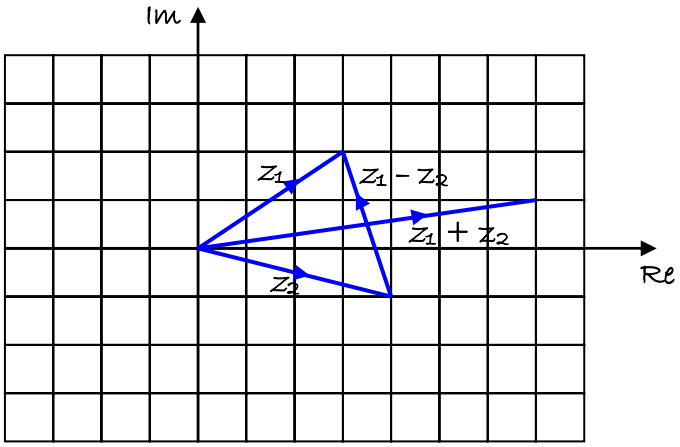
EdExcel Further Pure 1

Complex Numbers

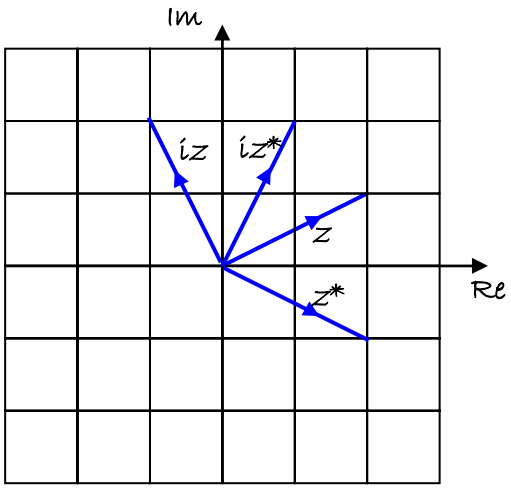
Section 2: The Argand diagram and the modulus-argument form

Solutions to Exercise

1.



2. $z = 2 + i$
$z^* = 2 - i$
$iz = 2i - 1 = -1 + 2i$
$iz^* = 2i + 1 = 1 + 2i$



(i) z^* is the reflection of z in the real axis
(ii) iz is a rotation of z through 90° anticlockwise.

3. (i) $z = -2\sqrt{3} - 2i$

$|z|^2 = \left(2\sqrt{3}\right)^2 + 2^2 = 12 + 4 = 16$

$|z| = 4$

$\arctan\left(\frac{2}{2\sqrt{3}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

z is in the 3rd quadrant so $\arg z = \frac{\pi}{6} - \pi = -\frac{5\pi}{6}$

(ii) $z = 1 - 3i$

$|z|^2 = 1^2 + 3^2 = 1 + 9 = 10$

$|z| = \sqrt{10}$

$\arctan\left(\frac{-3}{1}\right) = -1.25$ (3 s.f.)

z is in the 4th quadrant so $\arg z = -1.25$ (3 s.f.)

(iii) $z = -3 + 3i$

$|z|^2 = 3^2 + 3^2 = 9 + 9 = 18$

$|z| = 3\sqrt{2}$

$\arctan\left(\frac{1}{-1}\right) = -\frac{\pi}{4}$

z is in the 2nd quadrant so $\arg z = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$

4. (i) $3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 3\cos\frac{\pi}{4} + 3i\sin\frac{\pi}{4}$

$= 3 \times \frac{1}{\sqrt{2}} + 3i \times \frac{1}{\sqrt{2}}$

$= \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$

(ii) $6\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 6\cos\left(\frac{2\pi}{3}\right) + 6i\sin\left(\frac{2\pi}{3}\right)$

$= 6 \times -\frac{1}{2} + 6i \times \frac{\sqrt{3}}{2}$

$= -3 + 3i\sqrt{3}$

(iii) $2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = 2\cos\left(-\frac{\pi}{6}\right) + 2i\sin\left(-\frac{\pi}{6}\right)$

$= 2 \times \frac{\sqrt{3}}{2} + 2i \times -\frac{1}{2}$

$= \sqrt{3} - i$

5. (i) $z = 1 + 2i$

$|z| = \sqrt{1^2 + 2^2} = \sqrt{5}$

z is in the first quadrant, so $\arg z = \arctan\left(\frac{2}{1}\right) = 1.11$ (3 s.f.)

(ii) $z^* = 1 - 2i$

$|z^*| = \sqrt{1^2 + 2^2} = \sqrt{5}$

z^* is in the fourth quadrant, so $\arg z^* = \arctan\left(\frac{-2}{1}\right) = -1.11$ (3 s.f.)

(iii) $\frac{1}{z} = \frac{1}{1+2i} = \frac{1-2i}{(1+2i)(1-2i)} = \frac{1-2i}{5}$

$\left|\frac{1}{z}\right| = \frac{1}{5}\sqrt{1^2 + 2^2} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$

$\frac{1}{z}$ is in the fourth quadrant, so $\arg\frac{1}{z} = \arctan\left(\frac{-2}{1}\right) = -1.11$ (3 s.f.)

(iv) $\frac{1}{z^*} = \frac{1}{1-2i} = \frac{1+2i}{(1-2i)(1+2i)} = \frac{1+2i}{5}$

$\left|\frac{1}{z^*}\right| = \frac{1}{5}\sqrt{1^2 + 2^2} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$

$\frac{1}{z^*}$ is in the first quadrant, so $\arg\frac{1}{z^*} = \arctan\left(\frac{2}{1}\right) = 1.11$ (3 s.f.)

$\left|\frac{1}{z}\right| = \left|\frac{1}{z^*}\right| = \frac{1}{|z|} = \frac{1}{|z^*|}$

and $\arg z = \arg\frac{1}{z^*} = -\arg z^* = -\arg\frac{1}{z}$

6. (i) $|w| = 10$

$\arg w = \frac{\pi}{2}$

$w = 10\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$|z| = \sqrt{1+3} = 2$

z is in the first quadrant so $\arg z = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(ii) $|wz| = |w||z| = 10 \times 2 = 20$

$wz = 20\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$