

EdExcel Further Pure 1

Complex Numbers

Section 1: Introduction to complex numbers

Solutions to Exercise

1. (i) $z^2 + 25 = 0$

$$z^2 = -25$$

$$z = \pm 5i$$

(ii) $4z^2 + 9 = 0$

$$z^2 = -\frac{9}{4}$$

$$z = \pm \frac{3}{2}i$$

(iii) $z^2 - 2z + 2 = 0$

$$z = \frac{2 \pm \sqrt{4 - 4 \times 1 \times 2}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= 1 \pm i$$

(iv) $4z^2 + 4z + 5 = 0$

$$z = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times 5}}{8}$$

$$= \frac{-4 \pm \sqrt{-64}}{8}$$

$$= \frac{-4 \pm 8i}{8}$$

$$= -\frac{1}{2} \pm i$$

2. (i) (a) $z_1 + z_2 = 2 + 3i + 1 - 2i = 3 + i$

(b) $z_1 - z_2 = 2 + 3i - 1 + 2i = 1 + 5i$

(c) $z_1 z_2 = (2 + 3i)(1 - 2i) = 2 - 4i + 3i + 6 = 8 - i$

(d) $z_1^* = 2 - 3i$

(e) $z_2^* = 1 + 2i$

(f) $z_1^* + z_2^* = 2 - 3i + 1 + 2i = 3 - i$

(g) $z_1^* - z_2^* = 2 - 3i - 1 - 2i = 1 - 5i$

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$$(h) \quad z_1^* z_2^* = (2 - 3i)(1 + 2i) = 2 + 4i - 3i + 6 = 8 + i$$

$$(ii) \quad (a) \quad z_1 + z_2 = -2i + 3 + i = 3 - i$$

$$(b) \quad z_1 - z_2 = -2i - 3 - i = -3 - 3i$$

$$(c) \quad z_1 z_2 = -2i(3 + i) = -6i + 2 = 2 - 6i$$

$$(d) \quad z_1^* = 2i$$

$$(e) \quad z_2^* = 3 - i$$

$$(f) \quad z_1^* + z_2^* = 2i + 3 - i = 3 + i$$

$$(g) \quad z_1^* - z_2^* = 2i - (3 - i) = -3 + 3i$$

$$(h) \quad z_1^* z_2^* = 2i(3 - i) = 6i + 2 = 2 + 6i$$

$$z_1^* + z_2^* = (z_1 + z_2)^*$$

$$z_1^* - z_2^* = (z_1 - z_2)^*$$

$$z_1^* z_2^* = (z_1 z_2)^*$$

$$3. \quad z = (a + i)^4$$

$$= a^4 + 4a^3i + 6a^2i^2 + 4ai^3 + i^4$$

$$= a^4 + 4a^3i - 6a^2 - 4ai + 1$$

$$\text{If } z \text{ is real, } 4a^3 - 4a = 0$$

$$4a(a^2 - 1) = 0$$

$$4a(a + 1)(a - 1) = 0$$

$$\text{so } a = 0, -1 \text{ or } 1.$$

$$4. \quad (i) \quad \frac{1}{3 + 2i} + \frac{1}{3 - 2i} = \frac{3 - 2i + 3 + 2i}{(3 + 2i)(3 - 2i)}$$

$$= \frac{6}{9 + 4}$$

$$= \frac{6}{13}$$

$$(ii) \quad 3 + i + \frac{4}{3 - i} = 3 + i + \frac{4(3 + i)}{(3 - i)(3 + i)}$$

$$= 3 + i + \frac{4(3 + i)}{9 + 1}$$

$$= 3 + i + \frac{2}{5}(3 + i)$$

$$= \frac{7}{5}(3 + i)$$

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$$\begin{aligned} \text{(iii)} \quad \frac{3}{1-i} - \frac{2i}{2+i} &= \frac{3(1+i)}{(1-i)(1+i)} - \frac{2i(2-i)}{(2+i)(2-i)} \\ &= \frac{3+3i}{2} - \frac{4i+2}{5} \\ &= \frac{15+15i-8i-4}{10} \\ &= \frac{11+7i}{10} \end{aligned}$$

5. (i) One root is $2-i$ so the other root is $2+i$
Equation is $(z-2+i)(z-2-i)=0$

$$(z-2)^2 + 1 = 0$$

$$z^2 - 4z + 4 + 1 = 0$$

$$z^2 - 4z + 5 = 0$$

$$p = -4, q = 5$$

- (ii) One root is $1-3i$ so the other root is $1+3i$
Equation is $(z-1+3i)(z-1-3i)=0$

$$(z-1)^2 + 9 = 0$$

$$z^2 - 2z + 1 + 9 = 0$$

$$z^2 - 2z + 10 = 0$$

$$p = -2, q = 10$$

- (iii) One root is $2i$ so the other root is $-2i$
Equation is $(z-2i)(z+2i)=0$

$$z^2 + 4 = 0$$

$$p = 0, q = 4$$

- (iv) One root is $5-3i$ so the other root is $5+3i$
Equation is $(z-5+3i)(z-5-3i)=0$

$$(z-5)^2 + 9 = 0$$

$$z^2 - 10z + 25 + 9 = 0$$

$$z^2 - 10z + 34 = 0$$

$$p = -10, q = 34$$

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$$\begin{aligned} 6. \quad \frac{5}{a+bi} + \frac{2}{1+3i} &= 1 \\ \frac{5}{a+bi} &= 1 - \frac{2}{1+3i} = \frac{1+3i-2}{1+3i} = \frac{-1+3i}{1+3i} \\ \frac{a+bi}{5} &= \frac{1+3i}{-1+3i} \\ &= \frac{(1+3i)(-1-3i)}{(-1+3i)(-1-3i)} \\ &= \frac{-1-6i+9}{1+9} \\ &= \frac{8-6i}{10} \\ &= \frac{4-3i}{5} \\ a+bi &= 4-3i \end{aligned}$$