EdExcel Pure Mathematics 1

Complex Numbers

Topic assessment

1. Solve the equation $z^2 + 2z + 10 = 0$.
   Find the modulus and argument of each root. [5]

2. The complex number α is given by $\alpha = -2 + 5\text{i}$.
   (i) Write down the complex conjugate α^*. [1]
   (ii) Find the modulus and argument of α. [3]
   (iii) Find $\dfrac{\alpha + \alpha^*}{\alpha}$ in the form $a + b\text{i}$. [3]

3. Find the complex number z which satisfies $(2+\text{i})z + (3-2\text{i})z^* = 32$. [5]

4. (i) Given that $w = 1 + 2\text{i}$, express w^2, w^3 and w^4 in the form $a + b\text{i}$. [5]
   (ii) Given that w is a root of the equation $z^4 + pz^3 + qz^2 - 6z + 65 = 0$, find the values of p and q. [5]
   (iii) Write down a second root of the equation. [1]
   (iv) Find the other two roots of the equation. [6]

5. Complex numbers α and β are given by
   $$\alpha = 2\left(\cos\frac{\pi}{8} + \text{i}\sin\frac{\pi}{8}\right), \qquad \beta = 4\sqrt{2}\left(\cos\frac{5\pi}{8} + \text{i}\sin\frac{5\pi}{8}\right)$$
   (i) Write down the modulus and argument of each of the complex numbers α and β. Illustrate these two complex numbers on an Argand diagram. [3]
   (ii) Indicate a length on your diagram which is equal to $|\beta - \alpha|$, and show that $|\beta - \alpha| = 6$. [3]

6. (i) Show that $z_1 = 2 + \text{i}$ is one of the roots of the equation $z^2 - 4z + 5 = 0$.
   Find the other root, z_2. [3]
   (ii) Show that $\dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{4}{5}$. [3]
   (iii) Show also that $\text{Im}\,(z_1^2 + z_2^2) = 0$ and find $\text{Re}\,(z_1^2 - z_2^2)$. [4]

Total 50 marks

EdExcel FP1 Complex numbers Assessment solutions

Solutions to Topic assessment

1. $z^2 + 2z + 10 = 0$

Using the quadratic formula, $z = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 10}}{2}$

$$= \frac{-2 \pm \sqrt{-36}}{2}$$

$$= \frac{-2 \pm 6i}{2}$$

$$= -1 \pm 3i$$

$|-1 + 3i| = \sqrt{1^2 + 3^2} = \sqrt{10}$

(-1 + 3i) is in the second quadrant,

so $\arg(-1+3i) = \arctan\left(\dfrac{3}{-1}\right) + \pi = 1.89$ (3 s.f.)

$|-1 - 3i| = \sqrt{10}$

$\arg(-1-3i) = -1.89$

[5]

2. $\alpha = -2 + 5i$

(i) $\alpha^* = -2 - 5i$

[1]

(ii) $|\alpha| = \sqrt{2^2 + 5^2} = \sqrt{29}$

α is in the second quadrant, so $\arg\alpha = \pi + \arctan\left(\dfrac{5}{-2}\right)$

$$= \pi - 1.19$$

$$= 1.95 \text{ (3 s.f.)}$$

[3]

(iii) $\dfrac{\alpha + \alpha^*}{\alpha} = \dfrac{-2 + 5i - 2 - 5i}{-2 + 5i}$

$$= \frac{-4}{-2+5i}$$

$$= \frac{-4(-2-5i)}{(-2+5i)(-2-5i)}$$

$$= \frac{8 + 20i}{29} = \frac{8}{29} + \frac{20}{29}i$$

[3]

3. $(2+i)z + (3-2i)z^* = 32$

Let $z = x + iy$

$(2+i)(x+iy) + (3-2i)(x-iy) = 32$

$2x + ix + 2iy - y + 3x - 2ix - 3iy - 2y = 32$

$5x - 3y - ix - iy = 32$

Equating imaginary parts: $-x - y = 0 \Rightarrow y = -x$
Equating real parts: $5x - 3y = 32 \Rightarrow 5x + 3x = 32$
$\Rightarrow 8x = 32$
$\Rightarrow x = 4, \quad y = -4$

so $z = 4 - 4i$.

[5]

4\. (i) $w = 1 + 2i$

$w^2 = (1+2i)(1+2i) = 1 + 4i - 4 = -3 + 4i$

$w^3 = (-3+4i)(1+2i) = -3 - 2i - 8 = -11 - 2i$

$w^4 = (-11-2i)(1+2i) = -11 - 24i + 4 = -7 - 24i$

[5]

(ii) $z^4 + pz^3 + qz^2 - 6z + 65 = 0$

$-7 - 24i + p(-11-2i) + q(-3+4i) - 6(1+2i) + 65 = 0$

Equating real parts: $-7 - 11p - 3q - 6 + 65 = 0$

$11p + 3q = 52$ ①

Equating imaginary parts: $-24 - 2p + 4q - 12 = 0$

$p - 2q = -18$ ②

$2 \times ① + 3 \times ②$: $25p = 50 \Rightarrow p = 2, \quad q = 10$

[5]

(iii) A second root of the equation is $w^* = 1 - 2i$.

[1]

(iv) A quadratic factor is $(z - 1 - 2i)(z - 1 + 2i) = (z-1)^2 + 4$

$= z^2 - 2z + 1 + 4$

$= z^2 - 2z + 5$

$z^4 + 2z^3 + 10z^2 - 6z + 65 = (z^2 - 2z + 5)(z^2 + 4z + 13)$

The other two roots are the roots of the quadratic equation

$$z^2 + 4z + 13 = 0$$

$$z = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times 13}}{2}$$

$$= \frac{-4 \pm \sqrt{-36}}{2}$$

$$= \frac{-4 \pm 6i}{2}$$

$$= -2 \pm 3i$$

The other two roots are $-2 + 3i$ and $-2 - 3i$.

[6]

5\. (i) $\alpha = 2\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

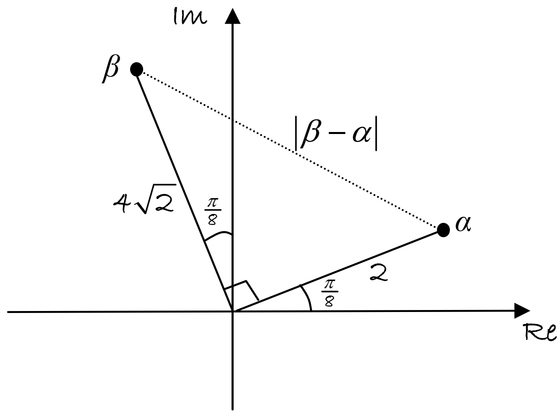
$|\alpha| = 2$

$\arg\alpha = \frac{\pi}{8}$

$\beta = 4\sqrt{2}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right)$

$|\beta| = 4\sqrt{2}$

$\arg\beta = \frac{5\pi}{8}$



[3]

(ii) The triangle is a right-angled triangle,

so $|\beta - \alpha|^2 = 2^2 + (4\sqrt{2})^2$

$= 4 + 32$

$= 36$

$|\beta - \alpha| = 6$

[3]

6\. (i) $z_1^2 - 4z + 5 = (2+i)^2 - 4(2+i) + 5$

$= 4 + 4i - 1 - 8 - 4i + 5$

$= 0$

The other root, $z_2 = z_1{}^* = 2 - i$

[3]

(ii) $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{2+i} + \frac{1}{2-i}$

$= \frac{2-i+2+i}{(2+i)(2-i)}$

$= \frac{4}{4+1}$

$= \frac{4}{5}$

[3]

(iii) $z_1^2 + z_2^2 = (2+i)^2 + (2-i)^2$

$= 4 + 4i - 1 + 4 - 4i - 1$

$= 6$

$\text{Im}\left(z_1^2 + z_2^2\right) = 0$

$z_1^2 - z_2^2 = (2+i)^2 - (2-i)^2$

$= 4 + 4i - 1 - 4 + 4i + 1$

$= 8i$

$\mathrm{Re}\left(z_1^2 - z_2^2\right) = 0$

[4]

