

AQA Further Pure 1

Complex Numbers

Topic assessment

1. Find the complex roots of the equation $x^2 + 6x + 10 = 0$ [2]

2. Express in the form $a + ib$:

(a) $(2 + i)^2$; [1]

(b) $(5 + 2i)(-3 + 4i)$. [1]

3. Given that $p, q \in \mathbb{R}$ find p and q where:

$$2(p + iq) = q - ip - 2(1 - i) \quad [2]$$

4. (a) Show that $(3 - i)^2 = 8 - 6i$. [1]

(b) The quadratic equation

$$az^2 + bz + 10i = 0$$

where a and b are real, has a root $3 - i$.

(i) Show that $a = 3$ and find the value of b . [6]

(ii) Determine the other root of the quadratic equation, giving your answer in the form $p + iq$. [3]

5. Given that $z = -2 + 2\sqrt{3}i$, show that $z^2 + 4z$ is real. [3]

6. If $z = x + iy$ and $z^* = x - iy$, solve $(z + 5) - 2(z^* - 3) = 1 - 2i$ [3]

7. If the roots of the equation $z^2 - (1 - i)z + 2 - i = 0$ are α and β ,

(a) find and simplify the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(b) Find the equation whose roots are $\alpha + 2\beta$ and $2\alpha + \beta$. [6]

Total 30 marks

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Solutions to Topic assessment

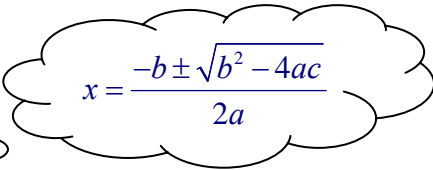
1. In $x^2 + 6x + 10 = 0$

$a = 1, b = 6$ and $c = 10$

$$x = \frac{-6 \pm \sqrt{36 - 4 \times 1 \times 10}}{2 \times 1}$$

$$x = \frac{-6 \pm 2i}{2} = -3 \pm i$$

The solutions are $x = -3 + i$ and $x = -3 - i$


$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

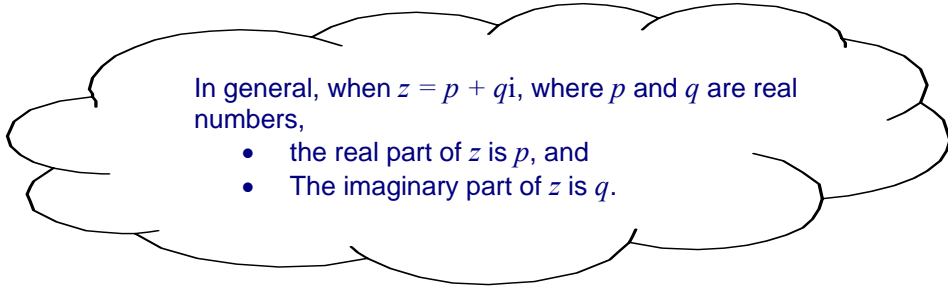
2. (a) $(2 + i)^2 = 4 + 4i + i^2 = 4 + 4i - 1 = 3 + 4i$

(b) $(5 + 2i)(-3 + 4i) = -15 + 20i - 6i + 8i^2$
 $= -15 + 14i - 8$
 $= -23 + 14i$



Remember $i \times i = -1$

3.



In general, when $z = p + qi$, where p and q are real numbers,

- the real part of z is p , and
- The imaginary part of z is q .

$$2(p + iq) = q - ip - 2(1 - i)$$

$$2p + 2iq = q - ip - 2 + 2i$$

Equating real parts:

$$2p = q - 2 \quad \text{①}$$

Equating imaginary parts:

$$2q = -p + 2 \text{ or } p = 2 - 2q \quad \text{②}$$

Substituting ② into ①:

$$2(2 - 2q) = q - 2$$

$$4 - 4q = q - 2$$

$$5q = 6$$

$$q = \frac{6}{5}$$

$$p = 2 - 2 \times \frac{6}{5} = -\frac{2}{5}$$

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4. (a) $(3-i)^2 = 9 - 6i + i^2 = 9 - 6i - 1 = 8 - 6i$

(b) (i) Since $z = 3 - i$ and $z^2 = 8 - 6i$,
therefore $a(8 - 6i) + b(3 - i) + 10i = 0$

Equating real parts: $8a + 3b = 0$ (1)

Equating imaginary parts: $-6a - b + 10 = 0$ (2)

Solving by (1) + 3 × (2) gives $-10a + 30 = 0$
 $a = 3$ and $b = -8$

(ii) Sum of roots $= -\frac{b}{a}$, in this case $\frac{8}{3}$

therefore the other root must be $\frac{8}{3} - (3 - i) = -\frac{1}{3} + i$.

5. $z = -2 + 2\sqrt{3}i$

$4z = -8 + 8\sqrt{3}i$

$z^2 = 4 - 8\sqrt{3}i - 12 = -8 - 8\sqrt{3}i$

so $z^2 + 4z = -8 - 8\sqrt{3}i - 8 + 8\sqrt{3}i = -16$

6. $(z + 5) - 2(z^* - 3) = 1 - 2i$

$(x + iy + 5) - 2(x - iy - 3) = 1 - 2i$

$-x + 11 + 3iy = 1 - 2i$

Equating real parts: $-x + 11 = 1 \Rightarrow x = 10$

Equating imaginary parts: $3y = -2 \Rightarrow y = -\frac{2}{3}$

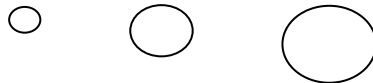
The solution is $z = 10 - \frac{2}{3}i$.

7. (a) In the quadratic equation $z^2 - (1 - i)x + 2 - i = 0$:

$a = 1, b = -(1 - i)$ and $c = 2 - i$

$\alpha + \beta = 1 - i$

$\alpha\beta = 2 - i$



Remember: When the quadratic equation $ax^2 + bx + c = 0$ has roots α and β then the product of the roots, $\alpha\beta = \frac{c}{a}$ and the sum of the roots, $\alpha + \beta = -\frac{b}{a}$.

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(b) For the new equation:

$$\begin{aligned}\text{Sum of roots} &= \alpha + 2\beta + 2\alpha + \beta \\ &= 3\alpha + 3\beta \\ &= 3(\alpha + \beta) \\ &= 3(1 - i)\end{aligned}$$

$$\begin{aligned}\text{Product of roots} &= (\alpha + 2\beta)(2\alpha + \beta) \\ &= 2\alpha^2 + \alpha\beta + 4\alpha\beta + 2\beta^2 \\ &= 2(\alpha^2 + \beta^2) + 5\alpha\beta \\ &= 2((\alpha + \beta)^2 - 2\alpha\beta) + 5\alpha\beta \\ &= 2(\alpha + \beta)^2 + \alpha\beta \\ &= 2(1 - i)^2 + 2 - i \\ &= 2 - 4i - 2 + 2 - i \\ &= 2 - 5i\end{aligned}$$

For the new equation, putting $a = 1$ gives $b = -3(1 - i)$ and $c = 2 - 5i$
so the new equation is $z^2 - 3(1 - i)z + 2 - 5i = 0$