Solution:

If a number when divided by 2 leaves remainder 1, it must be odd
Odd numbers can be written $2n+1$ or $2n-1$ where n is an integer.
If a number when divided by 3 leaves remainder 2, it can be written $3n+2$ or $3n-1$ where n is an integer.
If a number when divided by 4 leaves remainder 3, it can be written as $4n+3$ or $4n-1$ where n is an integer.
And so on...
So in each case the second description is more useful and we can quickly get a number which seems to do the trick,

$$2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 - 1$$

except this is bigger than 3000!!
But, for example, once we have $2 \times 3 \times 4 \times \underline{5} \times \ldots - 1$ then the number we are creating is automatically already of the form $10n-1$.
So all we need to take is the LCM of $2, 3, 4, \ldots, 10$.
$LCM(2,3,4,5,6,7,8,9,10) = 2^3 \times 3^2 \times 5 \times 7 = 2520$
So the first positive number which satisfies all the requirements is $2520 - 1 = 2519$
Any number which is a multiple of $2, 3, 4, \ldots, 10$ must be a multiple of 2520
So the numbers satisfying the requirements are precisely the numbers of the form $2520n - 1$ where n is any integer
[so infinitely many!]
(This could be solved using modular arithmetic and - I think! - the Chinese Remainder Theorem, but I need to clear the cobwebs for that!)