

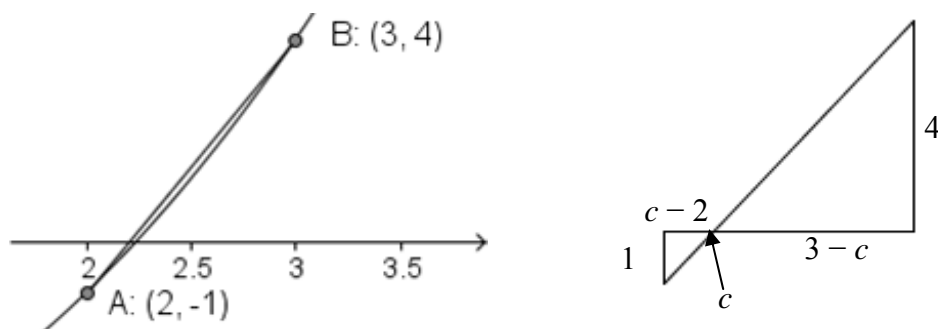
Trial and improvement

What you should know

How to solve equations by trial and improvement.
The properties of similar triangles.

New idea

The positive solution of $x^2 - 5 = 0$ is between $x = 2$ and $x = 3$ (because $2^2 - 5 = -1$, which is negative and $3^2 - 5 = 4$, which is positive). You can find an estimate for the solution by drawing a line between these two points on the curve and finding where it cuts the x -axis.



Using similar triangles the value of c can be found by solving $\frac{1}{c-2} = \frac{4}{3-c}$, which gives a value of $c = 2.2$.

Task: Linear interpolation

- Show that $\frac{1}{c-2} = \frac{4}{3-c}$ rearranges to give $c = 2.2$.
- Why is $\sqrt{5}$ the solution to $x^2 - 5 = 0$?
- How accurate is 2.2 as an estimate of $\sqrt{5}$?
- The solution to $x^2 - 5 = 0$ is between 2.2 and 3.
Apply the method again to find an improved estimate.

Take it further

- Find the solution to some other equations. Start with these.
 $x^2 - 10 = 0$ (Find the solution between $x = 3$ and $x = 4$.)
 $x^3 - 25 = 0$ (Find the solution between $x = 2$ and $x = 3$.)
- Find the solution to some equations you have already solved with trial and improvement. **NB:** The equation must be rearranged so that it equals 0.
How much quicker is this method? How much more accurate?
- How many times do you have to repeat it to get an answer accurate to 3 decimal places? (A spreadsheet may help you.)

Where this goes next

At A level this method, known as **linear interpolation**, is studied in Further Mathematics.